MEASUREMENT OF THE VISCOSITY OF EGGS BY THE USE OF A TORSION PENDULUM[1]

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EARLIER METHOD

During the spring of 1933 the problem of determining the viscosity of an egg from external data arose in connection with a study of other properties of eggs. Dr. Lyle Goodhue, formerly of the chemistry department of Iowa State College, suggested the possibility of using the damping of a torsion pendulum on which the egg was placed as a measure of this viscosity, and he designed apparatus to make these measurements. At his suggestion, the contents of eggs within shells of various sizes were removed and replaced by glycerol solutions of known viscosities. These eggs were then used to standardize the apparatus, the idea being that in this indirect way the viscosity of the egg contents could be measured. There are, however, three important difficulties with the method so far developed.

(1) An egg is not homogeneous in its interior, but consists of a central core structure composed of the yolk more or less firmly attached to the stiff albumen. This core structure floats in the highly fluid thin albumen which forms a lubricating layer inside the shell, so that measurements made by reference to a homogeneous model (i. e., the eggshell filled with glycerol solution) cannot be expected to be directly significant.

(2) The method employed is not complete enough to allow the apparatus to be easily standardized. No check was used to detect changes in the characteristics of the apparatus, and its physical properties were not given so that new apparatus could be produced which would yield the same results.

(3) Perhaps the most serious difficulty, and the one which initiated the present collaboration, is an ambiguity in the interpretation of the results of measurement with a torsion pendulum. If an eggshell is filled with a very thin liquid and the shell is oscillated in the torsion pendulum the inner portions of the liquid stand still and there is little damping of the pendulum. On the other hand, if the eggshell is filled with a very viscous liquid the egg will tend to turn as a solid body, and again there will be little damping of the pendulum. For each amount of damping there will be different viscosities which the contents of the egg can have. This result obtained by physical intuition is readily verified by experiment. Figure 1 shows the results of some experiments with an eggshell filled with glycerol solution. The coefficient of damping, K, of the torsion pendulum (calculated from experimental data in a way to be described later) is plotted against the Saybolt viscosity of the glycerol solutions. It is evident that for a given value of the coefficient K, there are two possible viscosities.

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The foregoing difficulties have caused the writers to change the method somewhat. In the first place, they have assumed a dynamical model of an egg which consists of a solid central portion to represent the yolk and firm albumen surrounded by a viscous fluid to represent the thin albumen.[2]

The second and third difficulties have been resolved by the use of the methods of mathematical physics. These results illustrate the use of mathematical-physical methods in biological problems as contrasted with the application of statistical mathematics. While the great utility of the statistical methods is generally recognized, the use of mathematics in its physical applications does not seem so well known in the biological field.

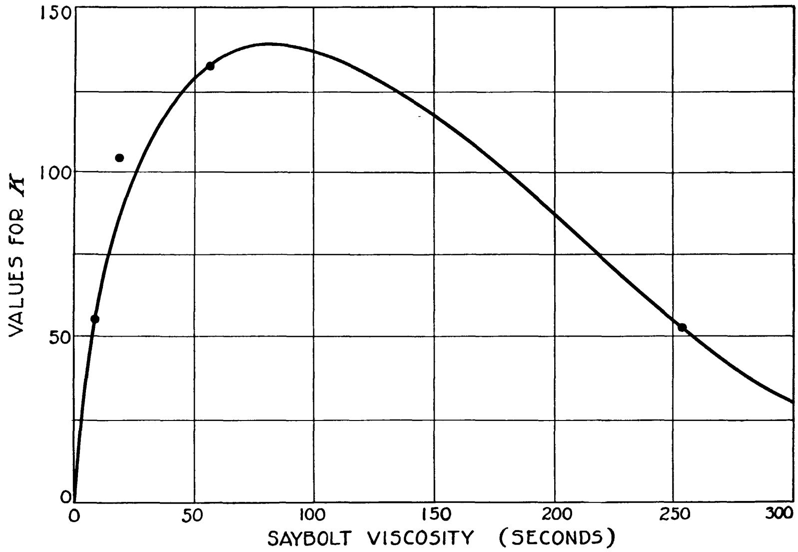

FIGURE 1.—Variation of *K* with Saybolt viscosity of eggshells containing glycerin.

EXPERIMENTAL METHOD

Figure 2 is a photograph of the apparatus used. A light metal frame, *f*, supports a horizontal hard rubber ring, *r*, in which the egg, *e*, may be placed with its small end downward; *b* is a brass rod upon which the weights, *w*, *w′*, slide to vary the moment of inertia of the frame. These weights are retained in a given position by setscrews. The frame is suspended between fixed supports by steel piano wire. A small mirror, *m*, attached to the upper end of the frame reflects a spot of light from the lamp house, *l*, onto a screen and enables one to follow the oscillations of the system.

When the system is slightly twisted and released it oscillates with an angular motion with the vertical wire supports as an axis. The period and frequency depend upon the moment of inertia of the system and the torsion constant of the wire supports. The period is deter-

[2] The existence of thin albumen enmeshed within the firm albumen has no significance in the present problem. Moreover, it is not necessary to take the fluidity of yolk into consideration, as a sufficiently accurate model can be obtained without this refinement.

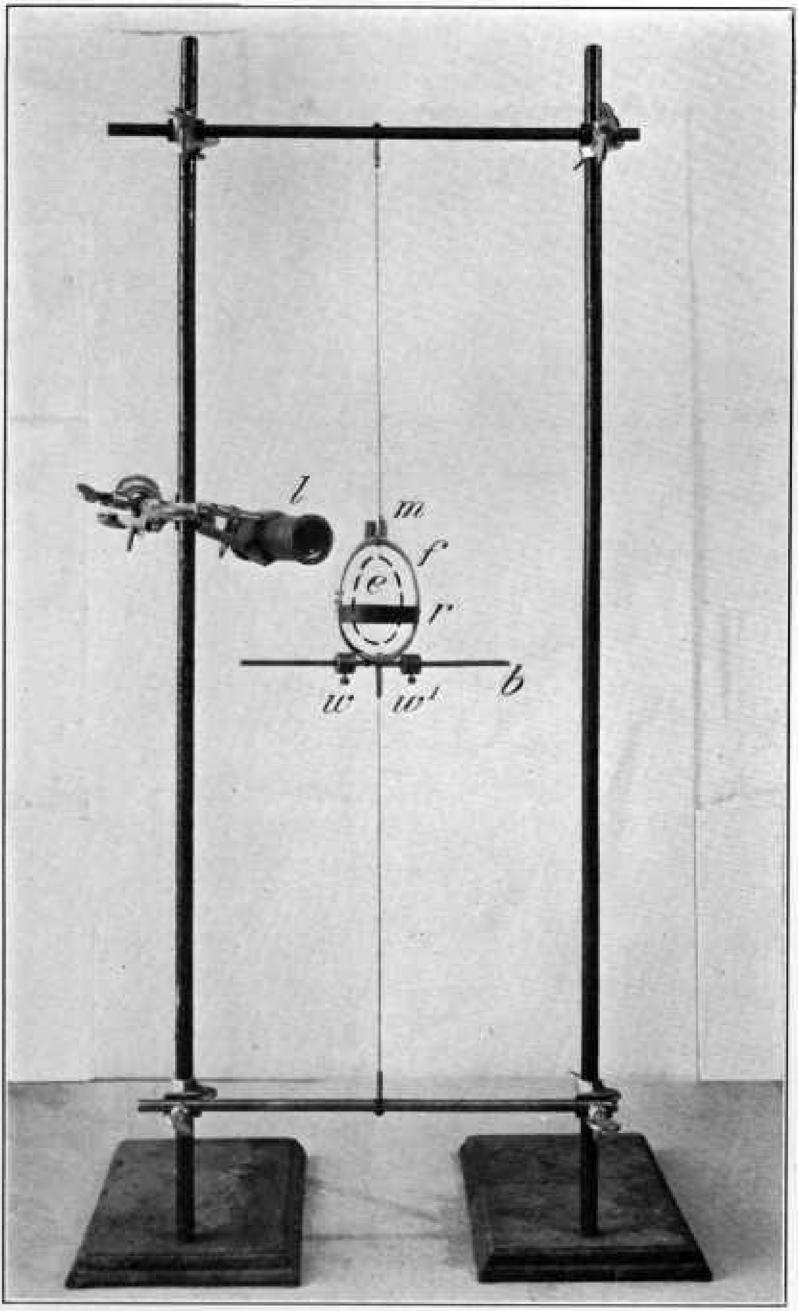

FIGURE 2.—Apparatus used in determining egg viscosity: *f*, Light metal frame; *r*, horizontal hard-rubber ing; *e*, the egg; *b*, brass rod; *w*, *w'*, weights; *m*, small mirror; *l* lamp.

mined by timing some convenient number of swings. It is, of course, observed that the angular amplitude decreases with time, and this decrease of amplitude is called the damping of the system. The damping is measured by taking either the number of swings or the time necessary for the angular amplitude to decrease from a certain value, θ', to a smaller value, θ'', as indicated by marks on the screen. The counting of swings has proved a more satisfactory method as it allows a mental interpolation if, as is usual, the extreme position of no single swing falls on the mark. Different observers doing this mental interpolation can generally interpolate at both marks and agree to within one- or two-tenths of a swing.

THEORETICAL BASIS

Let

θ=Angular position of frame from rest position.
I=Moment of inertia of frame.
μ=Torsion constant of wire.
c=Coefficient of damping resistance.
t=Time.

The differential equation of motion of the frame is $I\frac{d^2\theta}{dt^2}+c\frac{d\theta}{dt}+\mu\theta=0.$

It is convenient to divide this equation by I and use the new letters $l=\frac{c}{2I}$, $m^2=\frac{\mu}{I}$. In terms of these abbreviations the solution is $\theta=ae^{-lt}\cos(\omega t-\beta)$ in which $\omega=\sqrt{m^2-l^2}$, and a and β are arbitrary constants. The frequency, $f=\frac{\omega}{2\pi}$, and period, $P=\frac{2\pi}{\omega}$, are more often used in practical work than the angular frequency ω. The logarithmic damping $\delta=lP$ is easily seen to be the natural logarithm of the ratio of two successive amplitudes. If δ and I are small $\omega=\frac{2\pi}{P}=m=\sqrt{\frac{\mu}{I}}$. Now we shall adopt the convention that quantities measured with the empty frame go unmarked, that quantities measured with the bar of known moment of inertia, I', in the frame are marked with a bar above them while the presence of an egg will be denoted by a subscript on the quantities under consideration. Obviously $\bar{\mu}=\mu_1=\mu$. So from the last equation $\frac{2\pi}{\bar{P}}=\sqrt{\frac{\mu}{\bar{I}}}$, while of course $\frac{2\pi}{P}=\sqrt{\frac{\mu}{I}}$. Hence, we have $\frac{\bar{P}^2}{P^2}=\frac{\bar{I}}{I}$. But $\bar{I}=I+I'$ so $I=I'\frac{P^2}{\bar{P}^2-P^2}$. This relation is used to determine I.

An egg is represented by a central core structure of moment of inertia, J, surrounded by a viscous fluid which yields a torque T proportional to the difference of the angular velocities of the core $\frac{d\phi}{dt}$ and the shell $\frac{d\theta}{dt}$.

$$T=k\left(\frac{d\phi}{dt}-\frac{d\theta}{dt}\right)$$

When the egg is placed in the frame the resulting differential equations are easy to formulate and possible of solution, but this rigorous solution is intricate enough in form to be hard to use. A method of approximation is indicated by the fact that the moment of inertia of the egg is always small compared with that of the frame structure. So the reaction of the egg on the frame does not change its motion much except to increase its damping.

At first we assume the motion of the shell is purely sinusoidal, that is we may write $\theta = a \sin \omega t$. Now by Newton's law for rotary motion, $k(\theta - \phi) \text{-} J\ddot{\phi}$, in which J is the moment of inertia of the core of the egg and the dots denote differentiation with respect to time. Thus, $\dot{\phi} = \frac{d\phi}{dt}$ and $\ddot{\phi} = \frac{d^2\phi}{dt^2}$. Since θ is known we may easily solve this differential equation. The particular integral $\phi = \frac{ak^2}{k^2+J^2\omega^2} \sin \omega t - \frac{akJ\omega}{k^2+J^2\omega^2} \cos \omega t$ describes the sustained motion of the core structure which is needed for the present problem.

We may now calculate the work done by the shell on the contents of the egg during one cycle. It is $W = \int T d\theta = \frac{\pi k a^2 J^2 \omega^3}{k^2+J^2\omega^2}$. If the same method is applied to the normal damping torques, $T = c\dot{\theta}$, the work done per cycle is found to be $W = \int T d\theta = c\pi a^2 \omega$. So, if one writes $I\ddot{\theta} + c_1\dot{\theta} + \mu\theta = 0$ in which $c_1 = c + K$, $K = \frac{kJ^2\omega^2}{k^2+J^2\omega^2}$, one will have a differential equation furnishing nearly the correct solution for θ, for it will have the correct damping, and the egg is too small to affect the motion in any other way. We have chosen K as the measure of total egg viscosity.[3] It is easily calculated, in fact we have, $\delta = lP = \frac{c}{2I} P$. Likewise, $\delta_1 = l_1 P_1 = \frac{c_1 P_1}{2I}$. But the effect of an egg on P is too small to be important so that we may write $\delta_1 = \frac{c_1 P}{2I}$. So $K = c_1 - c = \frac{2I}{P}(\delta_1 - \delta)$. As before stated, δ is found by counting the number of swings needed for the pendulum to damp between two angular amplitudes θ' and θ''. The formula is $\delta = \frac{1}{n} \log \frac{\theta'}{\theta''}$. So we finally find $K = \frac{2I}{P}\left(\frac{1}{n_1} - \frac{1}{n}\right) \log_e \frac{\theta'}{\theta''}$. The angular amplitudes θ' and θ'' are determined by marks on the screen on which the spot of light moves. In this way $\log_e \frac{\theta'}{\theta''}$ becomes a constant of the apparatus.

The formula for K, $K = \frac{kJ^2\omega^2}{k^2+J^2\omega^2}$, is worth further study. If k be plotted against K we have a curve of the form shown in figure 3. The fact that there are two values of k for a given K and hence for a given damping of the torsion pendulum is clearly shown.

A plot of the variation of K with $\omega = 2\pi f$ is also instructive as it represents a relation that one may actually obtain in the measurement of a single egg. The relation of figure 3 cannot be thus realized, for the k of a given egg is a fixed quantity. However, before plotting the

[3] However, see the remarks about the relation of K and k on page 14.

equation we write it in the form $\frac{K}{k}=\frac{(J\omega/k)^2}{1+(J\omega/k)^2}$, that is, $Z=\frac{X^2}{1+X^2}$, $Z=\frac{K}{k}$, $X=\frac{J\omega}{k}$. This transformation of variables reduces the graph to

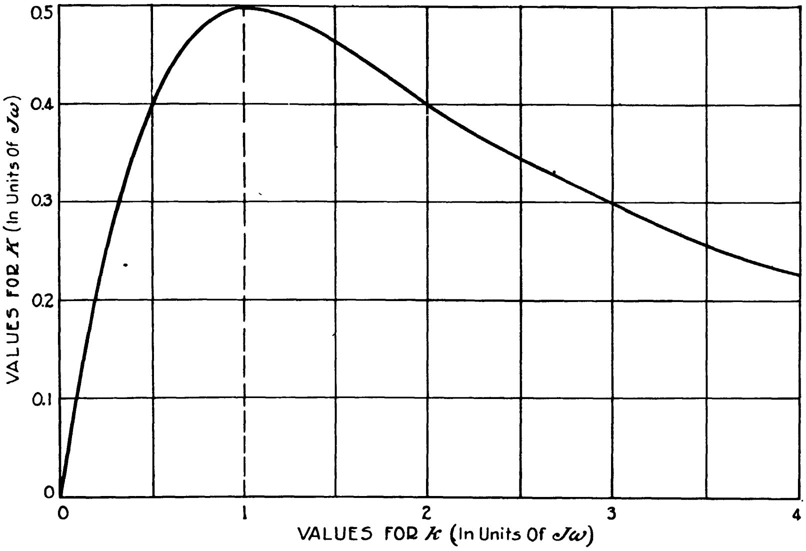

FIGURE 3.—Result of plotting k against K.

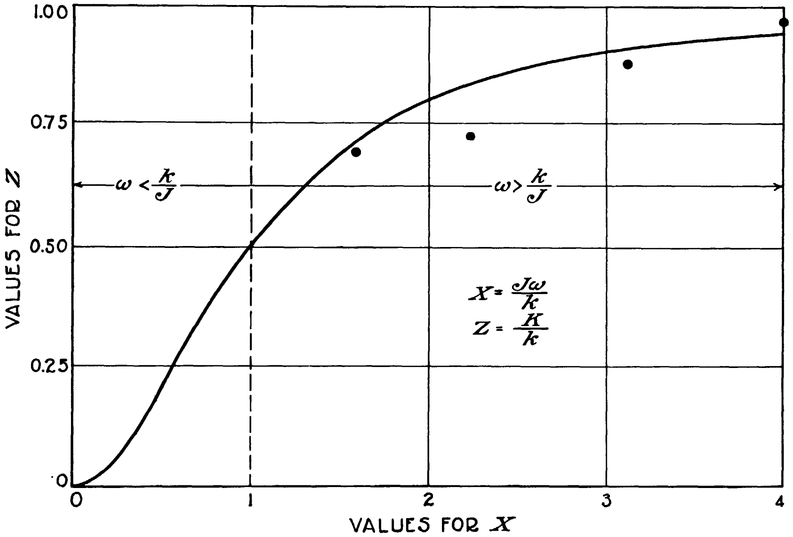

FIGURE 4.—Values for X and Z plotted after transformation of variables reducing the graph to a form applicable to any J and k.

a form applicable to any J and k. A glance at this graph (fig. 4) shows that as X increase Z approaches unity; that is, K approaches k.

DISCUSSION OF EXPERIMENTAL RESULTS

In a typical experimental run the damping effects of each of 12 eggs were measured at four different frequencies. The corresponding values of K were calculated and are presented in table 1.

TABLE 1.—*Variation of K with the frequency f at which it is measured*

f	ω	Data for egg no.—											
		1	2	3	4	5	6	7	8	9	10	11	12
0.675	4.25	131	65	74	72	175	67	129	71	111	63	82	73
1.01	6.33	104	70	72	88	212	73	124	82	82	61	71	73
1.43	8.97	115	85	86	97	190	88	122	97	97	72	81	81
1.83	11.5	113	93	93	97	158	92	128	106	105	83	85	87

It is rather clear that, in a general way, the variation of K with ω for most eggs fits the theoretical result indicated in figure 4. The points marked on the graph indicate the experimental results for the second egg when $J=$ 34 g, cm^2 and $k=97$. They agree well with the theoretical curve. The most troublesome feature of the experimental results is a tendency for K either to decrease or not to increase fast enough for low values of ω. This is strongly portrayed by the tabular results for eggs 1 and 9, and is present in eggs 3, 7, 10, and 11. This effect cannot be explained in terms of the model used but probably is caused by an initial breaking down of the egg structure as the experiment proceeds, causing the value of K to decrease as ω increases. As an example, the K value of egg no. 2863 was 347 when the determination was made at a frequency of 1.83 and at the amplitude given below. However, when this egg was severely shaken, and a second determination was made immediately after shaking, the K value had decreased to 136, and when this was repeated the K value dropped to 125. This indicates the desirability of treating the egg gently in the measuring process in order to avoid errors due to injury of the egg structure. This can be accomplished by conducting the measurements at small amplitude (i. e., small θ').

The writers used the values $\theta'=33°$ and $\theta''=23.7°$, but there is no objection to using smaller values, which would, of course, decrease the internal forces in the egg and prevent a partial disintegration of its structure.

The quantity K reaches its maximum value when $k=J\omega$ (fig. 3). As long as ω is maintained either above or below the value $\frac{k}{J}$ there is no question of an ambiguity in the value of k for a given K. For a reason to be mentioned presently the writers have seen fit to make the measurements in the range $\omega>\frac{k}{J}$, that is $X=\frac{J\omega}{k}>1$ (fig. 4). Except in these theoretical investigations of the method all measurements were made at the highest frequency, which makes X well above unity and in addition makes Z near unity, or K near k. This is the advantage of making the observations at relatively high frequency: Although K is given as the measure of egg viscosity, it is measured at a frequency which makes it nearly equal to the value of

k. This value of k will therefore depend only slightly on the exact frequency used. The other effects due to the measuring apparatus have of course been entirely eliminated.

In order to determine the relationship between the K values as determined by the torsion pendulum and commercial grading, comparisons were made between the K values and the two candling factors, yolk shadow and yolk movement, for a large number of eggs. For this purpose, all of the eggs from a number of pens were placed in a controlled temperature cabinet in the evening of the day on which they were laid, and the K values were determined the following day. Immediately after the K values were obtained the eggs were candled[4] and divided into five classes of yolk shadow and yolk movement ranging from light to dark for the shadow, and slow to fast for yolk movement.

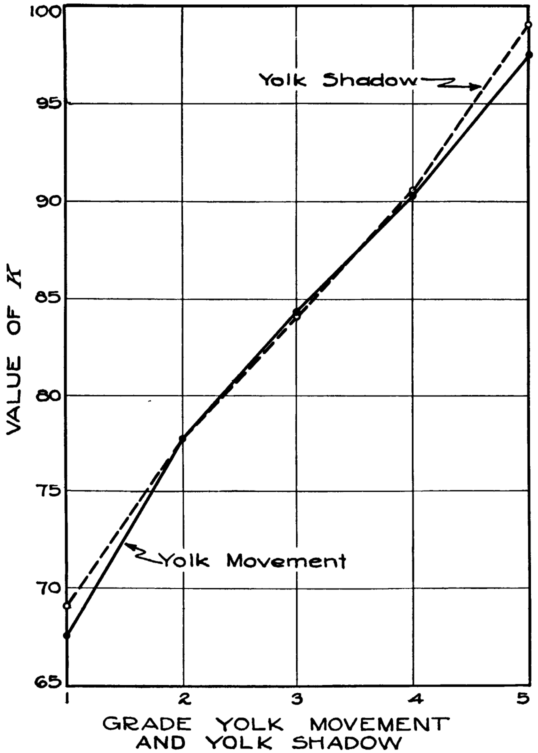

FIGURE 5.—Regressions of K on yolk movement and yolk shadow plotted for the five classifications of each of these two factors.

A total of 3,890 eggs was used in this study. The K values for the eggs were corrected for the period and moment of inertia of the torsion pendulum, and the number of swings to damp the empty pendulum. This was accomplished by means of the formula $K=\frac{2I(.329)}{P}\left(\frac{1}{N_1}-\frac{1}{N}\right)$, which is derived above. The regressions of K on yolk movement and yolk shadow were plotted for the five classifications for each of these two factors. These regression lines are shown in figure 5. There is a

[4] The classification of eggs into grades of yolk movement and yolk shadow was made by Dr. F. D. Perry, of Armour & Co.

very close relationship between yolk shadow and K value when the K value is not corrected for the weight of the egg; that is, in general, the eggs which show the most rapid yolk movement before the candle also exert the least damping effect upon the pendulum, and consequently yield the lower values for K. Similarly, those eggs which are classified as having a very distinct yolk shadow generally have the lower K values. It must be clearly understood, however, that these regression lines are only trend lines, and that individual eggs may vary considerably. This point is more clearly illustrated by the correlation coefficients for K and each of these candling factors. The correlation coefficient for K and yolk movement was found to be 0.2388 for this group of 3,890 eggs. For K and yolk shadow, the correlation coefficient was 0.2286. These correlation coefficients, while highly significant, are rather low, which may be accounted for by the large variation between individual eggs. Therefore, it may be concluded that the torsion pendulum method yields results which are in general agreement with results obtained by grading eggs before the candle; that is, that eggs which have a low total viscosity index are, in general, of poor quality as judged by yolk movement and yolk shadow. When individual eggs are studied, however, there is more variation in the results than would be desirable from an experimental viewpoint.